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QP CODE: 22101466

Reg No : .....

Name : .....

B.Sc DEGREE (CBCS) IMPROVEMENT / REAPPEARANCE EXAMINATIONS, MAY 2022

Fourth Semester

Core Course - MM4CRT01 - VECTOR CALCULUS, THEORY OF NUMBERS AND LAPLACE TRANSFORMS

(Common for B.Sc Computer Applications Model III Triple Main, B.Sc Mathematics Model I, B.Sc Mathematics Model II Computer Science)

2017 Admission Onwards

02B4286F

Time: 3 Hours Max. Marks : 80

Part A

*Answer any **ten** questions.*

*Each question carries **2** marks.*

1. Find an equation for the plane through $A(0,0,1)$, $B(2,0,0)$ and $C(0,3,0)$.
2. Find $\lim_{t\to\pi/4}\mathbf{r}(t)$ if $\mathbf{r}(t)=(\cos t)\mathbf{i}+(\sin t)\mathbf{j}+t\mathbf{k}$.
3. Define the curvature of a smooth plane curve. Show that, for any value of the parameter t, the curvature of the straight line is 0.
4. Define work done by a force over a curve in space.
5. Find the parametric equation of the cone $z=\sqrt{x^2+y^2}$, $z\geq 0$.
6. Define Curl of a vector field F.
7. Define the set of least non negative residues modulo n.
8. Define $absolute\ pseudoprimes$ with example.
9. Show that $18!\equiv -1 \pmod{437}$.
10. Find $\mathscr{L}^{-1}\left\{\frac{3}{s^2+2s}\right\}$.
11. Let $f(t)$ be continuous and satisfies the growth restriction for all $t\geq 0$. Also let $f'(t)$ be piecewise continuous on every finite interval on the semi-axis $t\geq 0$. Prove that the Laplace transform of $f'(t)$ satisfies $\mathscr{L}(f')=s\mathscr{L}(f)-f(0)$.
12. Evaluate the convolution $t*e^t$.

(10×2=20)

Part B

*Answer any **six** questions.*

*Each question carries **5** marks.*

13. Without finding $\mathbf{T}$ and $\mathbf{N}$, write the acceleration of the motion $\mathbf{r}(t) = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j},\ t > 0$ in the form $\mathbf{a} = a_{\mathbf{T}}\mathbf{T} + a_{\mathbf{N}}\mathbf{N}$.

14. Define the **tangent plane** and the **normal line** at a point on a smooth surface in space. Find the tangent plane and normal line of the surface $f(x, y, z) = x^2 + y^2 + z - 9 = 0$ at the point $P_0(1, 2, 4)$.

15. State Normal form and Tangent form of Green's Theorem.

16. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}, 0 \leq z \leq 1$.

17. Find the outward flux of the field $F = x^2 i + xzj + 3zk$ across the boundary of the sphere $x^2 + y^2 + z^2 \leq 4$.

18. Let a and b are integers that are not divisible by the prime p,then if $a^p \equiv b^p \pmod{p}$ prove that $a \equiv b \pmod{p}$.

19. If $\phi(n)|(n-1)$, prove that n is a square-free integer.

20. Using definition of Laplace Transform, prove that $\mathscr{L}(e^{at}) = \frac{1}{s-a}$. Hence find $\mathscr{L}(\cosh at)$.

21. State and prove first shifting theorem for Laplace Transform.

(6×5=30)

Part C

*Answer any **two** questions.*

*Each question carries **15** marks.*

22.
    1. Define the gradient vector of a function in the plane. Find an equation for the tangent to the curve $x^2 + y^2 = 4$ at the point $(\sqrt{2}, \sqrt{2})$.
    2. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $P_0(1, 1, 0)$ in the direction of $\mathbf{v} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$. In what direction does f change most rapidly at P_0, and what are the rates of change in these directions?

23. a) Define conservative vector field with an example.
    b) Show that the differential form in the integral (from $(1, 0, 0)$ to $(0, 1, 1)$) $\int (sinycosx)dx + (cosysinx)dy + dz$ is exact and evaluate the integral.

24. 1. State and prove Fermat's theorem.

2. Is the converse of Fermat's theorem is true or false? Give justifications.

25.

1. Using Laplace Transform, solve
$y'' + 2y' + 5y = 50\,t - 150,\ y(3) = -4,\ y'(3) = 14.$
2. Solve the Volterra integral equation of the second kind
$y(t) - \int_0^t y(\tau)\sin(t-\tau)d\tau = t.$

(2×15=30)